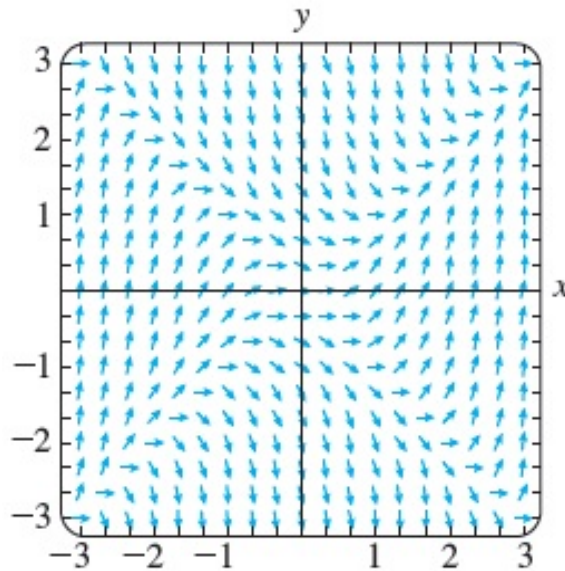


**DIRECTIONS** To receive full credit, you must provide complete legible solutions to the following problems in the space provided. Transfer all your answers to the space provided on the test paper

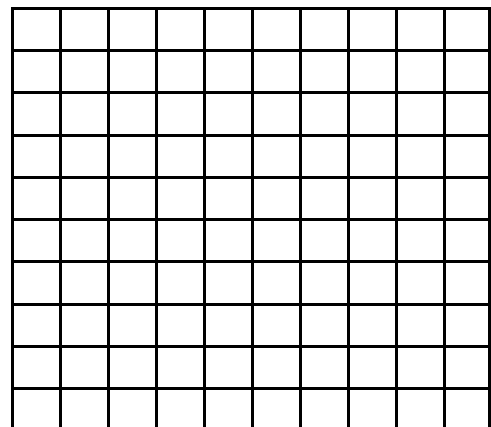
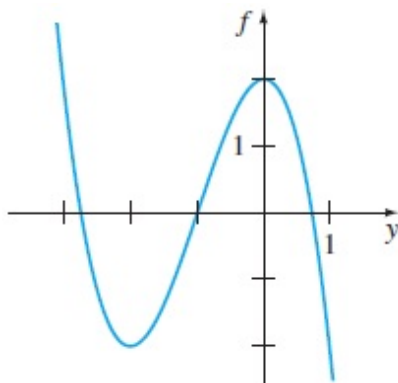
1. A computer-generated direction field for first order differential equation is given below. Sketch an approximate solution curve that passes through each of the indicated points.

a.  $(-2,1)$

b.  $(-1,2)$

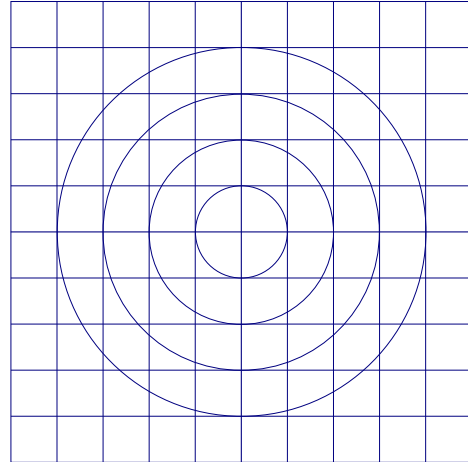


2. The given figure represents the graph of  $f(y)$ . Use the given grid to sketch a direction field over an appropriate grid for  $dy/dx = f(y)$



3. sketch isoclines  $f(x, y) = c$  for the given differential equation using the indicated values of  $c$ . Construct a direction field over a grid by carefully drawing lineal elements with the appropriate slope at chosen points on each isocline. In each case, use this rough direction field to sketch an approximate solution curve for the IVP consisting of the DE and the

initial condition  $y(0) = 1$ .  $\frac{dy}{dx} = x^2 + y^2; \quad c = \frac{1}{4}, c = 1, c = \frac{9}{4}, c = 4$



4. Consider the autonomous first-order differential equation  $dy/dx = y - y^3$  and the initial condition  $y(0) = y_0$ .
- Find the critical points and draw the phase line and phase portrait of the given differential equation.
  - Classify each critical point as asymptotically stable, unstable, or semi-stable. (List the critical points according to their stability)
  - Sketch the graph of a typical solution  $y(x)$  when  $y_0$  has the given values. Be sure to use the second derivative of  $y$  to determine the shape of the solution curves

$1 < y_0, \quad -1 < y_0 < -1, \quad y_0 < -1$

