

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Let  $H$  be the set of all vectors of the form

$$\begin{bmatrix} 2t \\ 0 \\ -t \end{bmatrix}, \text{ Show that } H \text{ is a subspace of } \mathbb{R}^3$$

2. Let  $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ ,  $v_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ ,  $v_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$ ,  $w = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ ,

- Is  $w$  in  $\{v_1, v_2, v_3\}$ ?
- How many linearly independent vectors in  $\{v_1, v_2, v_3\}$ ?
- Is  $w$  in the subspace spanned by  $\{v_1, v_2, v_3\}$ ? Prove your answer.

3. let  $W$  be the set of all vectors of the form shown, where  $a$ ,  $b$ , and  $c$  represent arbitrary real numbers. In each case, either find a set  $S$  of vectors that spans  $W$  or give an example to show that  $W$  is not a vector space.

$i.$   $\begin{bmatrix} 3a+b \\ 4 \\ a-5b \end{bmatrix},$        $ii.$   $\begin{bmatrix} -a+1 \\ a-6b \\ 2a+b \end{bmatrix}$

4. Determine if the set  $H$  of all matrices of the form  $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$  is a vector subspace of  $M_{2 \times 2}$ , where  $a$ ,  $b$  and  $c$  are real numbers.

5. Prove that All polynomials of degree less than or equal to three, with integers as coefficients form a vector subspace of the vector space of continuous functions on  $(-\infty, \infty)$