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NAME: KEY

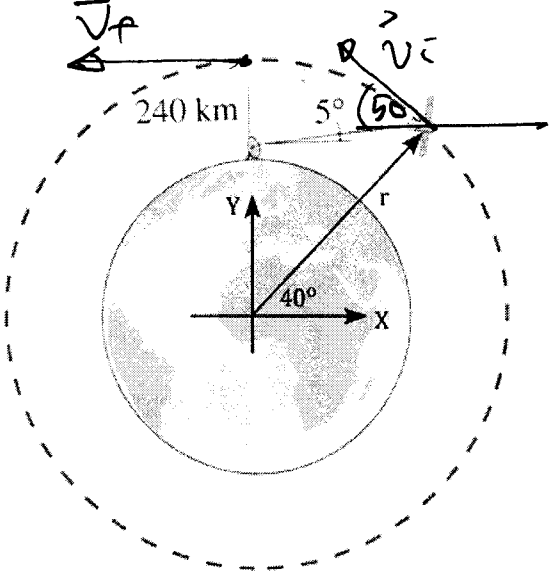
PHYSICS 4A
WINTER 2013
EXAM 1

PARTIAL CREDIT will be given so do what you can and make sure that you show all work for each problem. No credit will be given if no work is shown. The point value of each question is indicated.

240 km

1. A satellite is in a circular orbit ~~200~~ km above the earth's surface, moving at a constant speed of 9.6 km/s. A tracking station picks up the satellite when its 5.0° above the horizontal as shown below. The satellite is tracked until directly overhead. (R_E = 6380 km) (10 pts)

   a) Calculate the average acceleration in unit-vector notation during the tracking interval.
   b) Calculate the magnitude and direction of the instantaneous acceleration as is passes directly overhead.



$r = 6380\text{km} + 240\text{km}$

$\boxed{r = 6620\text{km}}$

$v_i = v_f = 9600\frac{m}{s}$

$\vec{v}_i = -9600\cos 50\,\hat{i} + 9600\sin 50\,\hat{j}$

$\vec{v}_f = -9600\,\hat{i}$

a) $\vec{a}_{ave} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$\Delta t = \dfrac{s}{v} = \dfrac{r\theta}{v} = \dfrac{(6620\times10^3\text{m})\left(50^\circ \times \frac{\pi}{180}\right)}{9600\frac{m}{s}}$

$\boxed{\Delta t = 602\text{s}}$

$\vec{a}_{ave} = \dfrac{-9600\,\hat{i} - (-9600\cos 50\,\hat{i} + 9100\sin 50\,\hat{j})\,m}{602\,s}$

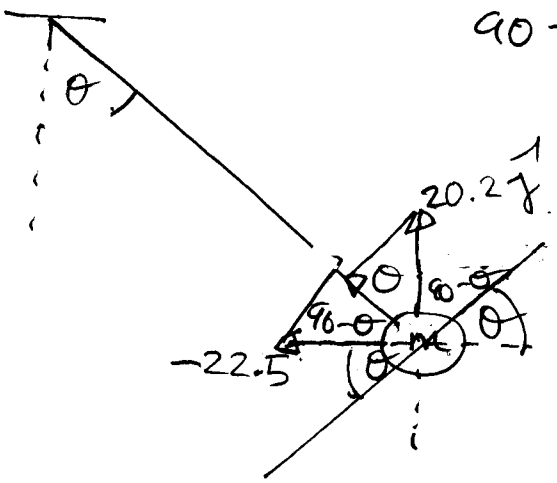
$\vec{a}_{ave} = \boxed{-5.7\,\hat{i} - 1.22\,\hat{j}}\ \frac{m}{s^2}$

b) $a_r = \dfrac{v^2}{r} = \dfrac{\left(9600\frac{m}{s}\right)^2}{6620\times10^3\,m} = \boxed{13.9\frac{m}{s^2}}$

2. A ball swings in a vertical circle at the end of a rope 2.0 m long. When the ball is 36.9° past the lowest point and on its way up, its total acceleration is **a** = (-22.5**i** + 20.2**j**) m/s^2. At that instant: (10 pts)
   a) Explain whether the ball is on its way up on the right side or on its way up on the left side.
   b) Sketch a vector diagram showing the components of this acceleration on the corresponding side.
   c) Determine the magnitude of its radial acceleration.
   d) Determine the speed and direction of the ball.

$\theta = 36.9^\circ$

$90 - \theta = 53.1^\circ$



a) & b)

$a_r = 22.5 \cos(90 - \theta) + 20.2 \cos\theta$

$a_r = 29.7 \frac{m}{s^2}$

c) $a_r = 29.7 \frac{m}{s^2}$

$a_t = 20.2 \sin\theta - 22.5 \sin(90 - \theta)$

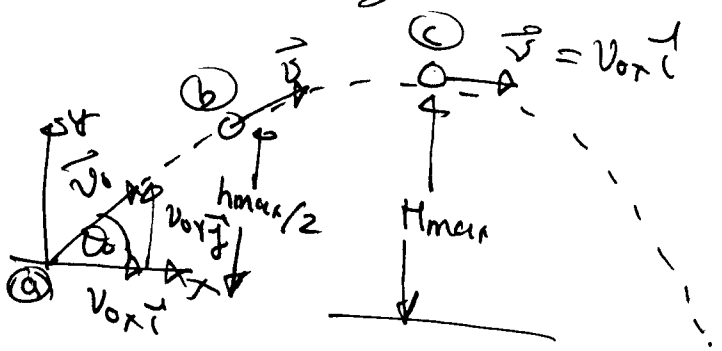
$a_t = -5.86 \frac{m}{s^2}$ slowing down since $a_t < 0$

d) $a_r = \frac{v^2}{r}$

$v = \sqrt{a_r r} = 7.7 \frac{m}{s}$ @ 36.9°

3. At ½ of its maximum height, the speed of a projectile is ¾ of its initial speed. Calculate the launch angle. (10 pts).

$v = \frac{3}{4} v_0$ @ $\frac{h_{max}}{2}$



$\theta_0 = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right)$

need v_{0y} & v_{0x} !

a → c

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$0 = v_{0y}^2 - 2g h_{max}$

$\boxed{v_{0y} = \sqrt{2g h_{max}}}$

At pt (b)

$v = \frac{3}{4} v_0$

$v^2 = \frac{9}{16} v_0^2$

$\boxed{v_{0x}^2 + v_y^2 = \frac{9}{16}(v_{0x}^2 + v_{0y}^2)}$

at $\frac{h_{max}}{2}$

a → b

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y^2 = 2g h_{max} + 2g \frac{h_{max}}{2}$

$\boxed{v_y^2 = g h_{max}}$

$\frac{7}{16} v_{0x}^2 = \frac{9}{16} v_{0y}^2 - v_y^2$

$= \frac{9}{16}(2g h_{max}) - g h_{max}$

$\frac{7}{16} v_{0x}^2 = \frac{1}{8} g h_{max}$

$\boxed{v_{0x} = \sqrt{\frac{2}{7} g h_{max}}}$

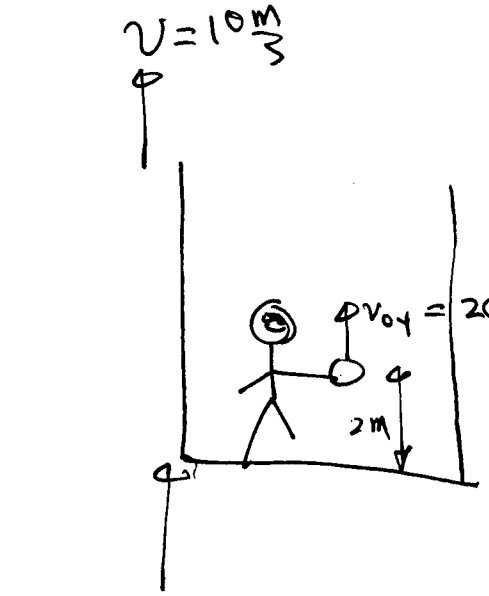
$\theta_0 = \tan^{-1}\sqrt{\frac{2g h_{max}}{\frac{2}{7} g h_{max}}} = \boxed{69.3^\circ}$

4. An elevator without a ceiling is speeding upward at a rate of 3.0 m/s^2. At the instant the elevator floor reaches a height of 28 above the ground and is moving at a speed of 10 m/s, a physics student inside the elevator throws a ball directly upward, from a height of 2 m above the elevator floor. The initial speed of the ball relative to the elevator is 20 m/s. (15 pts)
   a) What maximum height above the ground does the ball reach?
   b) Calculate the time it takes for the ball to return to the elevator floor in the earth's reference frame.
   c) Calculate the time it takes for the ball to return to the elevator floor in the elevator's reference frame.

a) $v_y^2 = v_{oy}^2 + 2a_y(y - y_o)$

$0 = (30)^2 + 2(-9.8)(H_{max} - 30)$

$H_{max} = 75.9\,m$



b) $y_f = y_b$

$y_{o_f} + v_{o_f}t + \frac{1}{2}a_f t^2 = y_{o_b} + v_{o_b}t + \frac{1}{2}a_b t^2$

$28 + 10t + 1.5t^2 = 30 + 30t - 4.9t^2$

$6.4t^2 - 20t - 2 = 0$

$t = 3.22\,s$

$y_f = y_b$

c) $0 = 2 + 20t + \frac{1}{2}(-9.8 - 3)t^2$

$0 = 2 + 20t - 6.4t^2$ ✓

$t = 3.22\,s$